A METHOD OF SOLVING SOME FUNCTIONAL EQUATIONS WITH THE ITERATES OF THE UNKNOWN FUNCTION

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The goal of the talk is to give a method of solving of a functional equation in which the unknown function occurs in the form of its own iterates. This method consists in replacing the above equation by other equation associated with it. The manner of solving both equations use the theory of (j,k)-symmetrical functions, which is included in paper [2].

Let k be an arbitrarily fixed positive integer, not smaller than 2, and let $\varepsilon = \exp(2\pi i/k)$. A subset U of the complex plane $\mathbb{C}$ will be called k-symmetrical set if $\varepsilon U = U$. The family of all k-symmetrical sets will be denoted by $\mathcal{S}_{\|}$.

Let D be an arbitrarily fixed nonempty subset of the complex plane $\mathbb{C}$. For a function $\varphi : D \to D$ and every positive integer n by $\varphi^{(n)}$ we will denote the n-th iterate of the function φ on the set D; in addition we assume that $\varphi^{(0)} = id_D$, (the identity mapping on D).

In the lecture we will consider the following functional equation

$$P(\varphi^{(0)}(z), \varphi^{(1)}(z), ..., \varphi^{(k-1)}(z)) = 0, \quad z \in D, \qquad (0.1)$$

where $P(z, x_0, x_1, \ldots x_{k-1})$ is a given polynomial of several complex variables $x_0, x_1, \ldots, x_{k-1}$ with the coefficients which are complex valued functions of one variable z, and φ is an unknown function.

The main our result consists in reducing the problem of solving equation of the form (0.1) to finding the solutions of the following functional equation

$$P(f(\varepsilon^0 w), f(\varepsilon^1 w), \ldots, f(\varepsilon^{k-1} w)) = 0, \quad w \in U, \qquad (0.2)$$

(with an $U \in \mathcal{S}_{\|}$ and the unknown function f) and to expressing the solution φ by the solution f. The method of solving equation (0.2) has been given in the recent papers [1] and [3].

[1] P. Liczberski, J. Polubinski, *On some functional equation*, Folia Scientiarum Universitatis Technicae Resoviensis, Mat. 17 (1995), 29-33.

[2] P. Liczberski, J. Polubinski, *On (j,k)-symmetrical functions*, Mathematica Bohemica, 120 (1995), 13-28.

[3] P. Liczberski, J. Polubinski, *On the applications of (j,k)-symmetrical functions to solving some functional equations*, to appear.