Differential –integral operators and the classes of functions defined by subordination

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Let U denote the unit disc in the complex plane and let A denote the class of functions analytic in U and satisfying the condition $f(0) = f'(0) - 1 = 0$.

Assume that $\lambda \in R$ and $\log(z-\zeta) \in R$ for $z \in U$, $\zeta \in U$, $z - \zeta > 0$. For a function $f \in A$ we define an operator

$$D_z^\lambda f(z) = \frac{1}{\Gamma(-\lambda)} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{1+\lambda}} d\zeta, \quad for \ \lambda < 0,$$

$$D_z^{m+\lambda} f(z) = \frac{1}{\Gamma(1-\lambda)} \frac{d^{m+1}}{dz^{m+1}} \int_0^z \frac{f(\zeta)}{(z-\zeta)^{\lambda}} d\zeta,$$

for $0 \leq \lambda < 1$, $m \in N \cup \{0\}$. Γ denotes the Gamma function. The multiplicity of $(z-\zeta)^{-(1+\lambda)}$ and $(z-\zeta)^{-\lambda}$ is removed by requiring $\log(z-\zeta) \in R$ when $z - \zeta > 0$. The above defined operator was introduced by Owa and Srivastava in 1978. With the aid of $D_z^\lambda f$ we define a following operator

$$\Omega^\lambda f(z) = \Gamma(2-\lambda) z^{\lambda-1} D_z^\lambda f(z).$$

For a fixed number $n \in N$ we denote

$$A(n) = \left\{ f : f \in A,\ f(z) = \sum_{k=n+1}^{\infty} a_k z^k,\ a_k \geq 0,\ k \geq n+1 \right\}.$$

Using the operator $\Omega^\lambda f$ we define a following subclass of $A(n)$:

$$V_\lambda(t,n,A,B) = \left\{ f \in A(n) : (1-t)\Omega^\lambda f(z) + t\Omega^{1-\lambda} f(z) \prec \frac{1+Az}{1-Bz} \right\},$$

where $A, B \in C$, $t \in R$, $n \in N$, $\lambda \neq -1, \pm 2, \pm 3, \ldots$. In this paper the necessary and sufficient condition for a function f to be in the class $V_\lambda(t,n,A,B)$ is determined and distortion theorems and radius of univalence are found.