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## Investigation of index of composition of entire functions

**ABSTRACT.** In this paper we generalize some of the known results about maximum modulus and maximum term of composition of entire function ([1], [2]).

### 1. INTRODUCTION

Let  $f(z) = \sum_{n=0}^{\infty} a_n z^n$  be entire function.  $\mu(r, f) = \max_{n \geq 0} \{a_n z^n\}$  is called the maximum term of  $f(z)$  on  $|z| = r$  and  $M(r, f) = \max_{|z|=r} |f(z)|$  - maximum modulus of  $f(z)$  on  $|z| = r$ . Sato [5] introduced the concept of "index" of the entire function.

$$\text{If } \limsup_{r \rightarrow \infty} \frac{\log^{[q]} M(r, f)}{\log r} = \rho_f(q), \quad 0 \leq \rho_f(q) \leq \infty \quad (1,1)$$

for  $q = 2, 3, \dots$ , where

$$\exp^{[0]} x = \log^{[0]} x = x, \quad \exp^{[m]} x = \log^{[-m]} x = \exp(\exp^{[m-1]} x) = \log(\log^{[-m-1]} x),$$

for  $\log^{[-m-1]} x > 0$ ,  $m = \pm 1, \pm 2, \dots$ , then  $f(z)$  is said to be of index  $q$  if  $\rho_f(q-1) = \infty$  and  $\rho_f(q) < \infty$ .

### 2. MAIN RESULTS

**Theorem.** Let  $f$  and  $g$  be two entire functions such that  $0 < \lambda_f(q) \leq \rho_f(q) < \infty$  and  $0 < \lambda_g(q) \leq \rho_g(q) < \infty$ . Then for every positive constant  $A$  and every real number  $x$

$$\lim_{r \rightarrow \infty} \frac{\log^{[2q-2]} M(r, f \circ g)}{\{\log^{[q]} M(r^A, f)\}^{1+x}} = \infty. \quad (3,1)$$

**Remark 1.** (i) For  $q=2$  and  $x=0$  this theorem is due to Singh and Baloria [1]

(ii) For  $A=1$ ,  $q=2$  and  $x=0$  this theorem is due to Song and Xang [3]

1. Singh A.P. and Baloria M.S., On the maximum modulus and maximum term of composition of entire functions, Indian J. pure appl. Math. 22(12) (1991) 1019-1026.
2. Sato D., On the rate of growth entire functions of fast growth, Bull. Amer. Math. Soc. 69(1963), 411-414
3. Song Guo-Dong and Yang Chung-Chun, Further growth properties of composition of entire and meromorphic functions, Indian J. pure appl. Math. 15(1), (1984), 67-82