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Tensor fields on LM induced by tensor fields on M by means of connections on M

We study how a classical linear connection ∇ on an m-dimensional manifold M and a tensor field t of type (r,s) on M can induce a tensor field $\mathcal{A}(\nabla,t)$ on the linear frame bundle LM over M. This problem is reflected in the concept of $\mathcal{M}f_m$-natural operators $\mathcal{A}: Q\times T^{(r,s)} \rightsquigarrow T^{(p,q)}L$ in the sense of [1]. We describe all natural operators $\mathcal{A}$ in question of finite order k.

Main Theorem. *Let S^k be the vector space of all k-jets at $0\in\mathbf{R}^m$ of classical linear connections ∇ on $\mathbf{R}^m$ given by the Christoffel symbols $\Gamma^i_{jl}:\mathbf{R}^m\to\mathbf{R}$ satisfying $\sum_{j,l=1}^m \Gamma^i_{jl}(x)x^jx^l=0$ for $i=1,...,m$. The space of all $\mathcal{M}f_m$-natural operators $\mathcal{A}: Q\times T^{(r,s)} \rightsquigarrow T^{(p,q)}L$ of order $k<\infty$ is free and finite dimensional module over the algebra of smooth maps $\mu: S^k\times J^k_0T^{(r,s)}\mathbf{R}^m\to\mathbf{R}$.*

In the proof of the main theorem we describe explicitly the module structure and construct explicitly the basis of this module.

[1] I. Kolář, P. W. Michor, J. Slovák, *Natural Operations in Differential Geometry*, Springer Verlag 1993.