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On bi-Lipschitz type inequalities for quasiconformal harmonic mappings

Let f be a sense-preserving homeomorphic self-mapping of the unit circle. According to the famous Radó-Kneser-Choquet theorem the Poisson extension F of f to the unit disk is a harmonic homeomorphic self-mapping of the unit disk. In 1997 and 1999 we established several results that provided intrinsic characterizations of f for F to be quasiconformal, in terms of the Cauchy and Cauchy-Stieltjes singular integrals involving f. In 2002 M. Pavlović then gave a decisive characterization of such mappings f. Moreover he proved that if F is quasiconformal, then F is bi-Lipschitz in the Euclidean metric. As improvements of this result, we find explicit estimations of bi-Lipschitz constants for any such quasiconformal harmonic mapping F that are expressed by means of the maximal dilatation K of F and $|F^{-1}(0)|$. Under the additional assumption $F(0) = 0$ the estimations are asymptotically sharp as $K \to 1$, so F behaves almost like a rotation for sufficiently small K.