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On the mappings of the unit disc onto disjoining domains

Let $U = \{z \in \mathbb{C} : |z| < 1\}$ and let $a_1, a_2,\ a_1 \neq a_2$, be the given complex numbers. By $M = M(a_1, a_2)$ we denote a family of all pairs of functions (f_1, f_2) regular in U and such that

$$f_1(0) = a_1, \quad f_2(0) = a_2, \qquad \text{and} \quad f_1(U) \cap f_2(U) = \emptyset.$$

Let $M^u,\ M^*,\ M^c$ denote the subclasses of M of functions which are univalent, starlike or convex respectively.

In this note we investigate the following functionals

$$I(f_1, f_2) = |f_1'(0)| + |f_2'(0)|,$$

$$K(f_1, f_2) = |f_1'(0) \cdot f_2'(0)|,$$

where $(f_1, f_2) \in M^*$ or $(f_1, f_2) \in M^c$.

For these functionals we obtain the estimations:

$$I(f_1, f_2) \leqslant 2|a_2 - a_1| \quad \text{for} \quad (f_1, f_2) \in M^c$$

and

$$K(f_1, f_2) \leqslant |a_2 - a_1|^2 \quad \text{for} \quad (f_1, f_2) \in M^c.$$

For $(f_1, f_2) \in M^u$, G. F. Bachtina obtained the estimation

$$I(f_1, f_2) \leqslant 4|a_2 - a_1|.$$

The functionals

$$I_{\alpha,\beta}(f_1, f_2) = \alpha|f_1'(0)| + \beta|f_2'(0)|, \quad \alpha \geqslant 0, \quad \beta \geqslant 0,$$

$$K_{\alpha,\beta}(f_1, f_2) = |f_1'(0)|^{\alpha} \cdot |f_2'(0)|^{\beta}, \quad \alpha \geqslant 0, \quad \beta \geqslant 0$$

are also investigated.