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On additive and multiplicative magic cubes

A *magic square* (a square array containing natural numbers $1, 2, \ldots, n^2$ such that the sum of the numbers along every row, column and diagonal is the same) has fascinated people for centuries. On the figure are depicted three square tables 3×3, 4×4 and 5×5 containing different natural numbers in such a way that the product of numbers in every row, column and diagonal is the same. Such tables are called *multiplicative magic squares.*

12	1	18
9	6	4
2	36	3

1	24	14	15
21	10	4	6
20	7	18	2
12	3	5	28

1	15	42	16	36
14	32	9	3	30
27	6	10	28	8
20	7	24	54	2
48	18	4	5	21

An additive magic cube is a natural generalization of a magic square. In 1686, the Polish mathematician *Adamas Kochansky* extended magic squares to three dimensions.

An *additive magic cube* of order n is a cubical array containing natural numbers $1, 2, 3, \ldots, n^3$ such that the sum of the numbers along every row and diagonal is the same, i.e. $\frac{n(n^3+1)}{2}$. By a row of a magic cube we mean an n-tuple of elements having the same coordinates on two places. Every additive magic cube of order n has exactly $3n^2$ rows and 4 diagonals. (Note: In [2] the following theorem is proved: *A magic p-dimensional cube of order n exists for every odd natural number n and every natural number p.*)

A *multiplicative magic cube* of order n is a cubical array containing n^3 mutually different natural numbers such that the product of the numbers along each row and every of its four diagonals is the same. We call this product *magic constant*. Until now we have no information about minimal magic product of a multiplicative magic cube (see [3]).

In our lecture we will give algorithms for making additive and multiplicative magic cubes. These topics and their applications can also by used in the teaching of mathematics at primary and secondary schools.

More information can be found in the following papers (see [5]).

[1] M. Trenkler, *A construction of magic cubes*, The Mathematical Gazette **84** 2000, 36–41.

[2] M. Trenkler, *Magic p-dimensional cubes, Acta Arithmetica* **96** *2001, 361–364.*

[3] *M.Trenkler, Súčinové magické štvorce a kocky (Multiplicative magic squares and cubes), Obzory matematiky, fyziky a informatiky* **31** *2002, 9–16.*

[4] *M.Trenkler, An algorithm for making magic cubes, The PME Journal (USA)* **12** *2005, 105–106.*

[5] *http://fedu.ku.sk/˜trenkler/papers.htm*